On considère la suite (I_n) *définie sur* $\mathbb{N}^*$ *par* $I_n = \int_0^{\frac{\pi}{4}} (\tan x)^n \, dx$.

2) a) Montrer que pour tout $n \in \mathbb{N}^*$, $I_n + I_{n+2} = \dfrac{1}{n+1}$

$$I_n + I_{n+2} = \int_0^{\frac{\pi}{4}} (\tan(x))^n \, dx + \int_0^{\frac{\pi}{4}} (\tan(x))^{n+2} \, dx$$

$$= \int_0^{\frac{\pi}{4}} \left(\tan^n(x) + \tan^{n+2}(x) \right) dx$$

$$= \int_0^{\frac{\pi}{4}} \tan^n(x) \left(1 + \tan^2(x) \right) dx$$

$\tan'(x) = 1 + \tan^2(x)$

$$= \int_0^{\frac{\pi}{4}} (\tan(x))' (\tan(x))^n \, dx$$

$u' u^n \to \dfrac{1}{n+1} u^{n+1}$

$$= \left[\frac{1}{n+1} \cdot (\tan(x))^{n+1} \right]_0^{\frac{\pi}{4}}$$

$$= \frac{1}{n+1} \left((\tan(\tfrac{\pi}{4}))^{n+1} - (\tan(0))^{n+1} \right)$$

$\tan(\frac{\pi}{4}) = 1$

$$= \frac{1}{n+1}$$

Donc $\boxed{I_n + I_{n+2} = \dfrac{1}{n+1}}$ $\forall n \in \mathbb{N}^*$